# ПАTIBIA UПIVERSITY OF SCIEПCE AПD TECHחOLOGY 

## FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES SCHOOL OF NATURAL AND APPLIED SCIENCES DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

| QUALIFICATION: BACHELOR OF ECONOMICS |  |
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| QUALIFICATION CODE: 07BECO | LEVEL: 5 |
| COURSE CODE: MFE511S | COURSE NAME: MATHEMATICS FOR ECONOMISTS 1A |
| SESSION: JULY 2023 | PAPER: THEORY |
| DURATION: 3 HOURS | MARKS: 100 |


| SECOND OPPORTUNITY/SUPPLEMENTARY EXAMINATION QUESTION PAPER |  |
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| EXAMINER | MR G. S. MBOKOMA, MRS A. SAKARIA |
| MODERATOR: | MR E. MWAHI |

## INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.
4. Decimal answers must be rounded to 4 decimals places

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Including this front page)

## QUESTION 1 (30 marks)

1.1 Determine the degree of the polynomial.

$$
\begin{equation*}
\left(9 x^{2} y^{3} z\right)^{2}-\frac{6 x^{2} y}{\left(y^{-3} z^{-2}\right)^{3}}+11 x^{4} y z^{6}+\left(4 x y^{2} z\right)^{3} \tag{3}
\end{equation*}
$$

1.2 Simplify the following expression

$$
\begin{equation*}
\text { 1.2.1 } 4 x^{2}-2 x(1+2 x)-2 x(1-y)-2 x y \tag{3}
\end{equation*}
$$

1.2.2 $\frac{1}{5}\left[\log \left(a^{2}+9 a\right)-\log (a+9)\right]$
1.2.3 $\frac{(p-q)^{-2 m n+n^{2}}}{(p-q)^{(n-m)^{2}}} \times(p-q)^{m^{2}}$
1.3 Solve the following indicial equation in $x:\left[\left(\frac{1}{20}\right)^{2+x} \times\left(\frac{1}{20}\right)^{8 x}\right]^{2}=1$
1.4 Evaluate $\lim _{x \rightarrow 2} \frac{x^{3}-4 x}{x-2}$
[5]
1.5 Use first principle of differentiation to evaluate $\frac{d y}{d x}$ if $y=x^{2}+1$

## QUESTION 2 ( 25 marks)

2.1 A total cost function is given as $C=\frac{a(b h+2)}{1+d h}$ where $a, b, d$, and $h$ are quantities produced. Make $h$ the subject of the formula and then evaluate $h$ when $a=20, c=10$, $d=1$ and $b=\frac{1}{4}$.
2.2 The Investment-Savings (IS) and Liquidity Preference - Money Supply (LM) models of a certain 3-sector economy, $Y=C+I+G$, economy compose the following:
$I S$
$L M$
$C=100+0.8 Y_{d} ; Y_{d}=Y-T$
$I=50-25 i$
$G=T=50$
$M^{d}=Y-25 i \ldots$. demand
$\frac{M^{s}}{P}=200 \ldots \ldots . .$. supply

Derive the $I S$ and $L M$ equations and hence determine the equilibrium levels of income and rate of interest, where $P=2$.
2.3 A firm uses labour ( L ) and machines $(\mathrm{K}$ ) to manufacture their products. The cost of labour is $\mathrm{N} \$ 40$ per unit and the cost of using a machine is $\mathrm{N} \$ 10$.

### 2.3.1 Derive the budget line of the firm.

2.3.2 Sketch a budget line for this firm, showing the combinations of (L,K) with total cost of $\mathrm{N} \$ 400$, label the budget line with $\left(C_{1}\right)$.
2.3.3 On the same graph, sketch another budget line with total cost of $N \$ 200$, label it with $\left(C_{2}\right)$
2.3.4 Discuss your observations between the two-budget lines.

## QUESTION 3 ( 25 marks)

3.1 A firm 's short-run production function is given by $Q=L e^{-0.02 L}$.
3.1.1 Find the marginal product of labour?
3.1.2 At $L=50$, determine whether the firm's maximes its production level?
3.1.3 What will be the production output at $L=50$ ?
3.2 The daily production function of a small-scale shoe manufacturer is given by $Q=\sqrt[3]{3 K^{2}+2 L^{3}}$, where $L$ is the labour input measured in daily work hours and $K$ is the cost of capital investment measured in thousands of dollars and $Q$ represents the daily production of shoes.

### 3.1.1 Determine the marginal productivity of capital and the marginal productivity of labour

3.1.2 Calculate the MRTS of the productions of shoes if workers put in 8 hours per day and cost of capital is $N \$ 4$.
3.3 Determine $\frac{d y}{d x}$, if $2 x^{3}-3 y^{2}+7 x y=0$

## QUESTION 4 (20 marks)

4.1 Determine the following integrals:
4.1.1 $\int\left(\frac{x^{3}+2 x^{2}-4 x+1}{\sqrt{x}}\right) d x$
4.1.2 $\int_{-2}^{3} e^{\frac{-x}{2}} d x$
4.2 The revenue and cost rates of a mining exploratory company are $R^{\prime}(t)=14-t^{\frac{1}{2}}$ and $C^{\prime}(t)=2+3 t^{\frac{1}{2}}$ respectively, where the time $t$ is measured in years and $R$ and $C$ are measured in millions of dollars.
4.2.1 How long should the exploration be continued to obtain the maximum profit? [4]
4.2.2 Calculate the maximum profit for this company.

